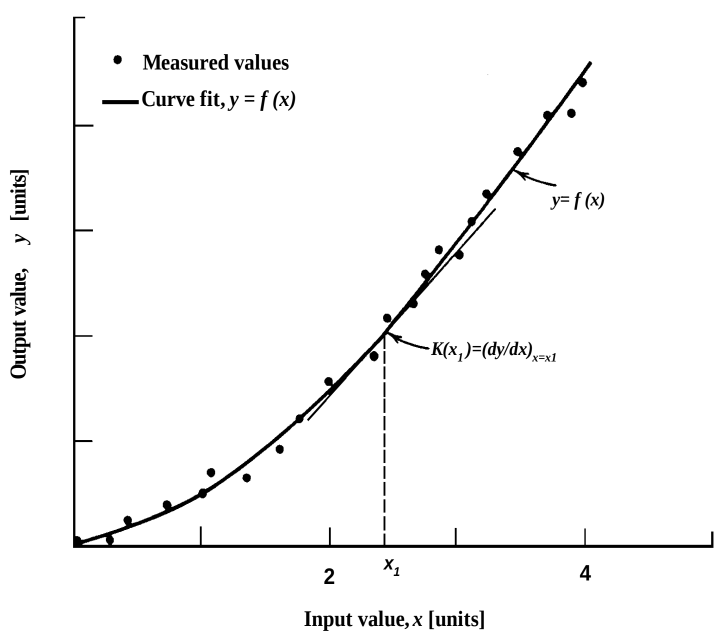
Some Notes on Device Calibration

The relationship between the input and the output of a measuring system is established during the calibration of a measuring system. A ***calibration*** is the act of applying a known value to the input of the measuring system for the purpose of observing the system output. The known value applied to the input is known as the ***standard***. By the application of a range of known values to the input and observation of the system output, a direct calibration curve can be developed for the measurement system. On such a curve the input x is plotted on the abscissa against the measurement output ***y*** on the ordinate as in Figure 1. In a calibration the input value should be a controlled input variable, while the measured output value becomes the dependent variable of the calibration.

Figure 1. A Static Calibration Curve



A calibration curve forms the logic by which a measurement system's indicated output can be interpreted during an actual measurement. For example, the calibration curve is the basis for fixing the output display scale on a measurement System, such as that of Figure 1. Alternatively, a calibration curve can be used as part of developing a functional relationship, an equation known as a ***correlation,*** between input and output. A correlation will have the form $y = f(x)$ and is determined by applying physical reasoning and curve fitting techniques to the calibration curve. The correlation can then be used in later measurements to ascertain the unknown input value based on the output value, the value indicated by the measurement system.

Static Calibration

The most common type of calibration is known as a *static calibration.* In this procedure, a known value is input to the system under calibration and the system output is recorded. The term "static" refers to a calibration procedure in which the values of the variables involved remain constant during a measurement, that is, they do not change with time. In static calibrations, only the magnitudes of the known input and the measured output are important.

A representative static calibration curve is shown in Figure 1. The measured data points describe the static input-output relationship for a measurement system. A polynomial curve fit to the data may be conveniently used to describe this relationship as $y = f(x)$.

Dynamic Calibration

In a broad sense, dynamic variables are time dependent in both their magnitude and frequency content. The input-output magnitude relation between a dynamic input signal and a measurement system will depend on the time-dependent content of the input signal. When time-dependent variables are to be measured, a dynamic calibration is performed in addition to the static calibration. A ***dynamic calibration*** determines the relationship between an input of known dynamic behavior and the measurement system output. Usually, such calibrations involve either a sinusoidal signal or a step change as the known input signal.

Static Sensitivity

The slope of a static calibration curve yields the ***static sensitivity*** *of* the measurement system. As depicted graphically in the calibration curve of Figure 1, the static sensitivity, $\boldsymbol{K}$, at any particular static input value, say x_1 , is evaluated by

$$K = K(x_i) = \left(\frac{dy}{dx}\right)_{x=x_1}$$

where $\boldsymbol{K}$ is a function of $\boldsymbol{x}$. The static sensitivity is a measure relating the change in the indicated output associated with a given change in a static input. Since calibration curves can be linear or nonlinear depending on the measurement system and on the variable being measured, $\boldsymbol{K}$ may or may not be constant over a range of input values.

Range

The proper procedure for calibration is to apply known inputs ranging from the minimum and to the maximum values for which the measurement system is to be used. These limits define the operating *range* of the system. The input operating range is defined as extending from $x_{\min}$ *to* $x_{\max}$. This range defines its ***input span*** expressed as the difference between the range limits $r_i = x_{\max} - x_{\max}$

Similarly, the output operating range is specified from $y_{\min}$ to $y_{\max}$. The output span or ***full-scale-operating range*** *(FSO)* is expressed as $r_o = y_{\max} - y_{\max}$

It is important to avoid extrapolation beyond the range of known calibration during measurement since the behavior of the measurement system is uncharted in these regions. As such, the range of calibration should be carefully selected.

Accuracy

The accuracy of a system can be estimated during calibration. If we assume that the input value is known exactly, then the known input value can be called the true value. The *accuracy* of a measurement system refers to its ability to indicate a true value exactly. Accuracy is related to absolute error. ***Absolute error, e,*** is defined as the difference between the true value applied to a measurement system and the indicated value of the system:

$$e = true.value - indicated.value$$

from which the percent relative accuracy is found by

By definition, accuracy can be determined only when the "true value" is known, such as during a calibration.

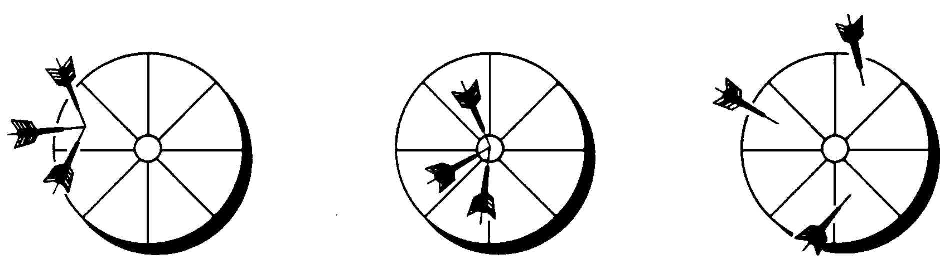
$$A = \left(1 - \frac{|e|}{true.value}\right) * 100$$

An alternative form of calibration curve is the deviation plot. Such a curve plots the difference or deviation between a true or expected value, y', and the indicated value, y, versus the indicated value. Deviation curves are extremely useful when the differences between the true and the indicated value are too small to suggest possible trends on direct calibration plots. They are often required in situations that require errors to be reduced to the minimums possible.

Precision and Bias Errors

The repeatability or precision of a measurement system refers to the ability of the system to indicate a particular value upon repeated but independent applications of a specific value of input. ***Precision error*** is a measure of the random variation to be expected during such repeatability trials. An estimate of a measurement system precision does not require a calibration, per se. <u>But note that a system that repeatedly indicates the same wrong value upon repeated application of a particular input would be considered to be very precise regardless of its known accuracy.</u>

Figure 2. Illustrating precision, bias errors & accuracy



(a) High repeatability gives low precision error but gives no direct indication of accuracy

(b) High accuracy means low precision and bias errors

(c) Bias & precision errors lead to poor accuracy

The average error in a series of repeated calibration measurements defines the error measure known as *bias.* Bias error is the difference between the average and true values. Both precision and bias errors affect the measure of a system's accuracy.

The concepts of accuracy, and bias and precision errors in measurements can be illustrated by the throw of darts. Consider the dart board of Figure 2 where the goal will be to throw the darts into the bull's-eye. For this analogy, the bull's-eye can represent the true value and each throw can represent a measurement value. In Figure 2a, the thrower displays good precision (i.e., low precision error) in that each throw repeatedly hits the same spot on the board, but the thrower is not accurate in that the dart misses the bull's-eye each time. This thrower is precise, but we see that low precision error alone is not a measure of accuracy. The error in each throw can be computed from the distance between the bull's-eye and each dart. The average value of the error yields the bias. This thrower has a bias to the left of the target. If the bias could be reduced, then this thrower's accuracy would improve. In Figure 2b, the thrower displays high accuracy and high repeatability, hitting the bull's-eye on each throw. Both throw scatter and bias error are near zero. High accuracy must imply both low precision and bias errors as shown. In Figure 2c, the thrower displays neither high precision nor accuracy with the errant throws scattered around the board. Each throw contains a different amount of error. While the bias error is the average of the errors in each throw, precision error is related to the varying amount of error in the throws. The accuracy of this thrower's technique appears to be biased and lacking precision. The precision and bias errors of the thrower can be computed using statistical methods. Both precision and bias errors quantify the error in any set of measurements and are used to estimate accuracy.

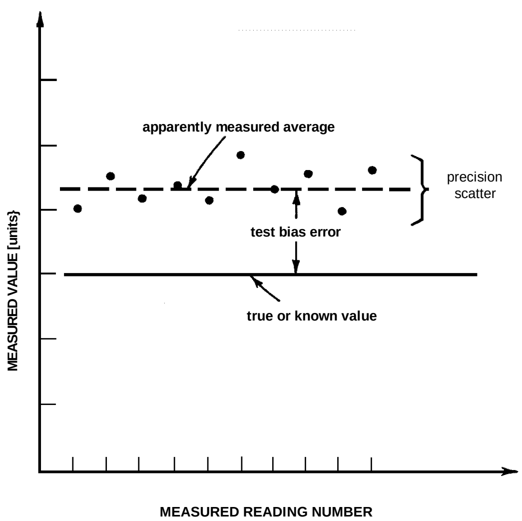


Figure 3. Effect of Precision & Bias errors on Calibration Readings

Suppose a measurement system was used to measure a variable whose value was kept constant and known exactly, as in a calibration. Ten independent measurements are made with the results, Figure 3. The variations in the measurements, the observed scatter in the data, would be related to the system precision error: associated with the measurement of the variable. That is, the scatter is mainly due to (1) the measurement system and (2) the method of its use, since the value of the variable is essentially constant. However, the offset between the apparent average of the readings and the true value would provide a measure of the bias error to be expected from this measurement system.

In any measurement other than a calibration the error cannot be known exactly since the true value is not known. But based on the results of a calibration, the operator might feel confident that the error is within certain bounds (a plus or minus range of the indicated reading). Since the magnitude of the error in any measurement can only be estimated, one refers to an estimate of the error in the measurement as the ***uncertainty*** present in the measured value. Uncertainty is brought about by errors that are present in the measurement system, its calibration, and measurement technique, and is manifested by measurement system bias and precision errors.

The precision and bias errors of a measurement system are the result of several interacting errors inherent to the measurement system, the calibration procedure, and the standard used to provide the known value. These errors can be delineated and quantified as elemental errors through the use of particular calibration procedures and data reduction techniques. An example is given for a typical pressure transducer in Table 1.

TABLE 1. Manufacturer's Specifications: Typical Pressure Transducer

Operation:	Input range	0 to 1000 cm (FSO)
	Excitation	+/- 15_V dc
	Output range	0 to 5 v dc
Performance:	Linearity error	+/- 0.5% full scale (FSO)
	Hysteresis error	< +/- 0.15% full scale (FSO)
	Sensitivity error	+/- 0.25% of reading
	Thermal sensitivity error	+/- 0.02% /degC of reading
	Thermal zero drift	0.02% /degC full scale (FSO)

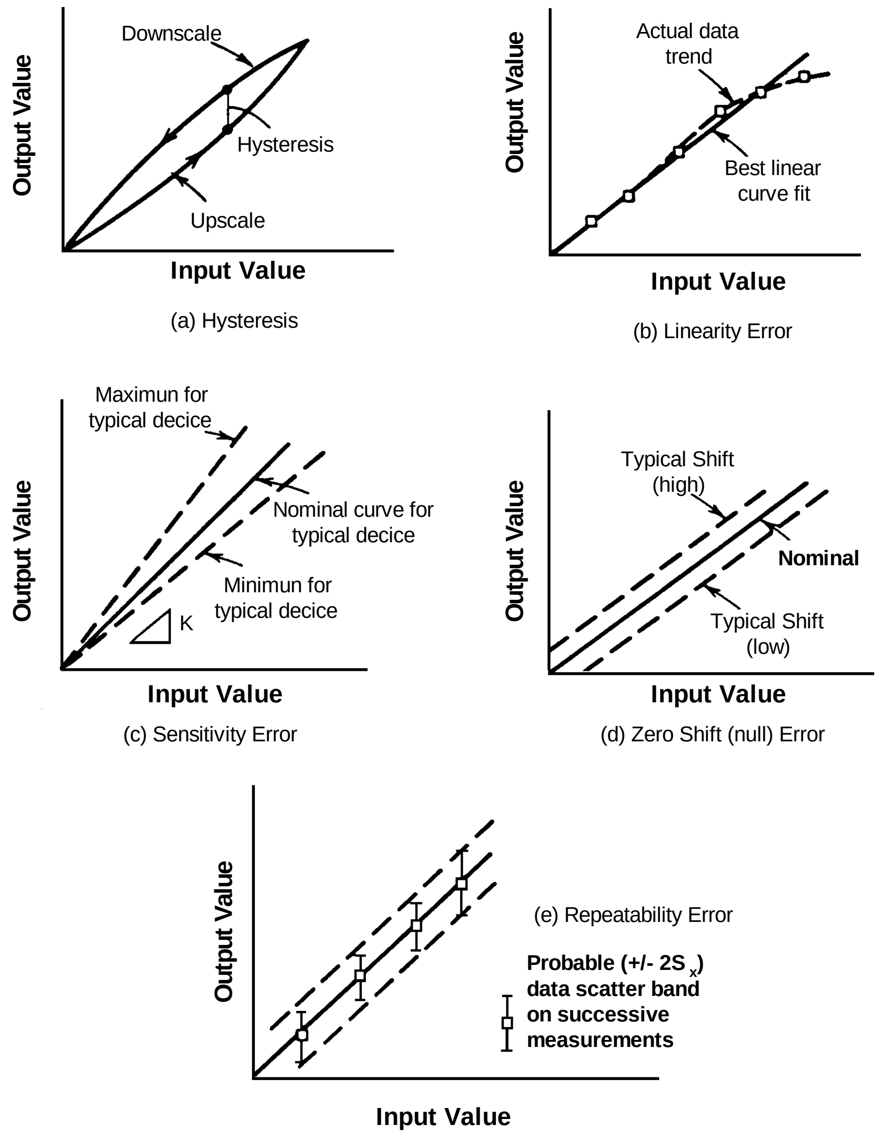


Figure 4. Examples of the Elements of Error

Sequence Calibration

A *sequence calibration* applies a sequential variation in the input value over the desired input range. This may be accomplished by increasing the input value (up-scale direction) or by decreasing the input value (downscale direction) over the full input range.

Hysteresis

The sequence calibration is an effective diagnostic technique for identifying and quantifying; hysteresis error in a measurement system. ***Hysteresis error*** refers to differences between an upscale sequence calibration and a downscale sequence calibration. The hysteresis error of the system is given by $e_h = (y)_{upscale} - (y)_{downscale}$, The effect of hysteresis in a sequence calibration curve is illustrated in Figure 4a. Hysteresis is usually specified for a measurement system in terms of the maximum hysteresis error as a percentage of full-scale output range (FSO):

$$\%(e_h) = \frac{[e_h(x)]_{\max}}{r_o} * 100$$

such as that indicated in Table 1. Hysteresis occurs when the output of a measurement system is dependent on the previous value indicated by the system. Such dependencies can be brought about through some realistic system limitations such as friction or viscous damping in moving parts or residual charge in electrical components. Some hysteresis is normal for any system and affects the precision of the system.

Random Calibration

A random calibration applies a randomly selected sequence of values of a known input over the intended calibration range. The random application of input tends to minimize the impact of interference. It breaks up hysteresis effects and observation errors. It ensures that each application of input value is independent of the previous. This reduces calibration bias error. Generally, such a random variation in input value will more closely simulate the actual measurement situation.

A random calibration provides an important diagnostic test for the delineation of several measurement system performance characteristics based on a set of random calibration test data. In particular, linearity error, sensitivity error, zero error, and instrument repeatability error, as illustrated in Figure 4*e,* can be quantified from a static random calibration

Linearity Error

Many instruments are designed to achieve a linear relation between an applied static input and indicated output values. Such a linear static calibration curve would have the general form:

$$y_L(x) = a_o + a_1 x$$

where the curve fit $y_L(x)$ provides a predicted output value based on a linear relation between $\boldsymbol{x}$ and $\boldsymbol{y}$. However, in real systems, truly linear behavior is only approximately achieved. As a result, measurement device specifications usually provide a statement as to the expected linearity of the static calibration curve for the device. The relation between $y_L(x)$ and measured value $y(x)$ is a measure of the nonlinear behavior of a system:

$$e_L(x) = y(x) - y_L(x)$$

where $e_L(x)$ is the ***linearity error*** that arises in describing the actual system behaviour by eq.1. Such behavior is illustrated in Figure 4b in which a linear curve has been fitted through a calibration data set. For a measurement system that is essentially linear in behavior, the extent of possible non-linearity in a measurement device is often specified in terms of the maximum expected linearity error as a percentage of full-scale output range:

$$\%(e_L)_{\max} = \frac{[e_l(x)]_{\max}}{r_o} * 100$$

This value is listed as the linearity error expected from the pressure transducer in Table 1. Statistical methods of quantifying such data scatter about a line exist.

Sensitivity and Zero Errors

The scatter in the data measured during a calibration affects the precision in the slope of the calibration curve. As shown for the linear calibration curve in Figure 4*c,* if we fix the zero intercept at zero (a zero output from the system for zero input), then the scatter in the data leads to precision error in estimating the slope of the calibration curve. The ***sensitivity error,*** e_K **,** is a statistical measure of the precision error in the estimate of the slope of the calibration curve. The static sensitivity of a device is also temperature dependent and this is often specified. In Table 1., the sensitivity error reflects calibration results at a constant reference ambient temperature, whereas the thermal sensitivity error was found by calibration at different temperatures.

If the zero intercept is not fixed but the sensitivity is constant, then drifting of the zero intercept introduces a vertical shift of the calibration curve, as shown in Figure 4d. This shift of the zero intercept of the calibration curve is known as the ***zero error****,* e_Z of the measurement system. Zero error can usually be reduced by periodically adjusting the output from the measurement system under a zero in-put condition. However, some random variation in the zero intercept is common, particularly with electronic and digital equipment subjected to temperature variations (e.g., thermal zero-drift in Table 1.).

Instrument Repeatability

The ability of a measurement system to indicate the same value upon repeated but independent application of the same input is known as the ***instrument repeatability****.* Specific claims of repeatability are based on multiple calibration tests (replication) performed within a given lab on the particular unit. Repeatability, as shown in Figure 4e, is based on a statistical measure called the standard deviation, S_x , a measure of the variation in the output for a given input. The value claimed is usually in terms of the maximum expected error as a percentage of full-scale output range:

$$\%(e_r)_{\max} = \frac{2(S_x)}{r_o} * 100$$

The instrument repeatability reflects only the error found under controlled calibration conditions. It does not include the additional errors introduced during measurement due to variation in the measured variable or due to procedure.

Reproducibility

The term "**reproducibility**" when reported in instrument specifications, refers to the results of separate repeatability tests. Manufacturer claims of instrument reproducibility must be based on multiple repeatability tests (replication) performed in different labs on a single unit.

Instrument Precision

The term "**instrument precision**" when reported in instrument specifications, refers to the results of separate repeatability tests. Manufacturer claims of instrument precision must be based on multiple repeatability tests (replication) performed in different labs on different units of the same manufacture.

Overall Instrument Error

An estimate of the ***overall instrument error*** is made based on all known errors. This error is often misleadingly referred to as the ***instrument accuracy*** in some instrument specifications. An estimate is computed from the square root of the sum of the squares of all known errors. For $\boldsymbol{M}$ known errors, the instrument error, $\boldsymbol{e}$, is estimated by

$$e_I = \left[e_1^{\,2} + e_2^{\,2} + \ldots\ldots\ldots\ldots e_M^{\,2}\right]^{1/2}$$

For example, for an instrument having known hysteresis *(h)*, linearity *(L)*, sensitivity *(K)*, and repeatability *(R)* errors, the instrument error is estimated by

$$e_I = \left[e_h^{\,2} + e_L^{\,2} + e_K^{\,2} + e_R^{\,2}\right]^{1/2}$$

End note.

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